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Journal of Sound and Vibration 301 (2007) 576–591
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Dynamic characteristics of eddy current dampers and couplers

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Received 6 January 2005; received in revised form 16 October 2006; accepted 16 October 2006
Available online 11 December 2006

Abstract

Eddy current dampers are promising devices for the passive, semi-active and regenerative control of vibrations. The aim of the present paper is to model the dynamic behavior of eddy current dampers and couplers and, by extension, of resistively shunted synchronous motors with permanent magnets. The modeling approach uses Faraday's law for the computation of the eddy current; the torque is then computed from the Lorentz force acting on the conductors. The electromechanical model is valid under rather general conditions and can be interfaced to the model of a mechanical structure to describe the coupled behavior. The relevant equations are solved for constant speed, and for a small amplitude torsional vibration of the rotor. The constant speed operation is characterized by the torque to speed curve which is typical of eddy current couplers and induction machines. The torsional vibration operation is characterized by the mechanical impedance and is typical of eddy current dampers. The torque to speed curve and the mechanical impedance are influenced by the same parameters. The relation between them constitutes a valuable design tool as it allows one to convert the steady-state characteristics into a dynamic model and vice versa.

1. Introduction

Eddy current dampers and couplers are based on the interaction between a conductor and a magnetic field in relative motion. Due to Faraday's law, the motion determines an electric field in the conductor, this induces eddy currents that interact with the magnetic field. The interaction generates Lorentz forces that act to oppose the relative motion [1]. As the force is proportional to the relative speed, the principle has been exploited in a variety of dampers. The same working principle is exploited in induction machines where a rotating (or translating) magnetic field generated by the stator induces currents in the rotor; the induced currents interact with the magnetic field of the stator to generate a torque or a force on the moving part. Due to its nature an induction machine generates a mechanical action (torque or force) only if the moving part and the magnetic field have different speeds (slip speed); a certain amount of power is therefore dissipated in the process.

Eddy current devices can also be used as viscous joints to transmit a torque between two shafts. In this case they are referred to as "joints" or as "couplers" in the mechanical or in the electrical machines literature, respectively.

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.10.015

Nomenclature

$\mathbf{B}_s$	magnetic flux density vector of the stator
c_{em}	damping coefficient in equivalent mechanical system
c_0	ratio between torque and speed for low constant speed
d	diameter of the rotor
$F_{1,2}$	Lorentz force on the conductors of windings 1 and 2
GBP	gain-bandwidth product
$i_{r1,2}$	current flowing in the rotor coils 1,2
i_r	rotor current in complex coordinates (relative to the rotor)
i_r^*	rotor current in complex coordinates (relative to the stator)
$i_{r\omega\Omega}$	rotor current due to torsional oscillations (for coupler)
k_{em}	stiffness in equivalent mechanical system
l	rotor length
L_r	coil inductance
$\mathbf{n}_{s1,2}$	unit vectors of the reference frame fixed to the stator
$\mathbf{n}_{r1,2}$	unit vectors perpendicular to the plane windings 1 and 2
N	number of rotor coils
R_r	coil resistance
s	Laplace variable
S	surface on the rotor
T	external torque applied to the rotor
T_{em}	electromagnetic torque acting on the rotor
T_Ω	torque due to constant angular speed (for coupler)
$T_{\omega\Omega}$	torque due to torsional oscillations (for coupler)
V_Ω	voltage due to constant angular speed (for coupler)
$V_{\omega\Omega}$	voltage due to torsional oscillations (for coupler)
Z_m	mechanical impedance
$\phi_{r1,2}$	total magnetic flux linked to coils 1,2
$\phi_{rs1,2}$	stator magnetic flux linked to coils 1,2
ϕ_{rs0}	magnetic flux of the stator linked to the rotor
θ	angular position of the rotor
$\theta_{1,2}$	angular position of coils 1,2
ω	angular frequency
ω_p	pole frequency
Ω	rotor angular speed (if constant)

Subscript

em	electromagnetic
r	rotor
rs	rotor to stator
s	stator

From the point of view of the system architecture, induction machines are very similar to eddy current dampers or couplers. The main difference is that electromagnets are usually adopted in induction machines to generate the magnetic field, permanent magnets are instead more common in dampers and couplers. Rare earth, high remanent magnetic field, permanent magnets allow one to build light machines without electrical connections to energize the electromagnets.

The analysis of induction machines is usually based on the magnetostatic assumption. The geometric configuration of such machines permits a bidimensional analysis in terms of the magnetic vector potential, either using the finite element method or integrating analytically the Maxwell's equations. Nagaya and Kojima [2–4] analyze the interaction between planar conducting elements of various shapes and a constant magnetic field perpendicular to the conductor. The study is performed considering the conductor as purely resistive, the force to velocity characteristic is then linear regardless of the velocity. This is valid if the excitation frequency is negligible compared to the resistive-inductive time constant.

In addition to the case of permanent magnets and solid conductors in relative motion, eddy current devices may be based on the generation of eddy currents in a stationary electrical circuit. The electromotive force is generated in this case by the time-varying magnetic flux linking the circuit [5,6]. The devices based on the motion of a conductor are referred by Graves et al. [6] as "motional emf devices". The devices based on the variable reluctance are referred as "transformer emf devices". The same authors compare the viscous damping coefficient that can be obtained by the two types of dampers. Similarly to Nagaya, Graves et al. do not take the inductive contribution into account. Vance et al. [7] investigate the application of motional and transformer dampers for aircraft engine applications. In order to obtain a variable damping, they adopt an electromagnet to generate the magnetic field of the stator. Their experimental results show that the damping coefficient of motional dampers decreases with increasing vibration frequency.

Lequesne et al. [8] analyze a coupler consisting of a solid conductive disk rotating within the magnetic field produced by permanent magnets polarized axially. The main assumptions are that the relative speed between the conductor and the magnetic field is constant and that the fields are bidimensional. The inductive effect in the eddy current loops makes the torque to relative speed characteristic similar to that of an induction machine. For small speeds the characteristic is linear, after having reached a maximum it then decreases asymptotically to zero for high speeds. Lequesne et al. analyze the influence of the resistivity of the conductor (of non-ferromagnetic type) on that characteristic. The higher the resistivity, the smaller the initial slope, and the higher the speed corresponding to the maximum torque. The result is that the maximum torque is nearly independent from the resistivity.

Canova and Vusini [9] analyze a coupler similar to that of Lequesne. They solve the electromagnetic field equations for constant angular speed to obtain the torque to speed characteristics. For a given amount of permanent magnet and number of pole pairs they show that the conductor thickness can be optimized to maximize the electromagnetic torque. Similarly, the number of pole pairs can be optimized to obtain a maximum torque for a given conductor thickness and relative speed. Ferromagnetic materials are also considered for the conductor disk along with composite configurations including a layered conductor with a ferromagnetic core sandwiched between two non-ferromagnetic layers.

The assumption of a constant relative speed between the rotating magnetic field and the rotor is valid only for steady state or slow transient conditions. If the steady-state assumption is dropped, the literature shows two different approaches. The first is related to the analysis of induction motors during fast rotating speed transients [10,11]. The aim in this case is to study the effects of the transient on the electrical variables rather than on the mechanical ones. The second approach is focused on linear eddy current devices such as voice coil dampers and shunted electrodynamic actuators subject to small amplitude vibrations [1,12]. The aim in this case is to understand their frequency response.

Very seldom the frequency response or the mechanical impedance of torsional devices as eddy current dampers and couplers, shunted brushless and induction motors can be found in the literature [13]. Similarly, mixed operating conditions where torsional vibrations and steady-state motion occur at the same time cannot be found in the literature, even if they are of practical relevance for eddy current couplers.

The aim of the present paper is to study the dynamic behavior of eddy current dampers and couplers under dynamic conditions. The modeling approach is based on the computation of the electromechanical torque from basic principles, i.e. the induced currents and the Lorentz's forces. The equivalent circuit approach is deliberately avoided as it does not allow one to gain an immediate understanding of the electromechanical interaction.

The model is valid under rather general conditions, it is used to study the behavior for dampers subject to constant speed and to torsional vibrations. In the first case (constant speed) the torque is obtained as function of the relative speed (torque to speed characteristic). In the second (torsional vibrations) the characterization is performed in terms of mechanical impedance. The mechanical impedance is then related to the torque to speed characteristic [8,9] that is commonly obtained from the commercial simulation tools and from the literature. The case of a coupler is also studied to determine its torsional behavior when the rotating speed oscillates about a non-zero value while the device is transmitting a mean mechanical power. The influence of the number of pole pairs is finally addressed to understand its influence as design parameter.

The present study offers two main contributions to the literature on eddy current dampers. The first is a simple electromechanical model of induction machines and eddy current dampers to study their behavior under rather general operating conditions. The second is the definition of a "conversion rule" to obtain the mechanical impedance from the torque to speed characteristic, and vice versa.

2. Electromechanical model of torsional eddy current dampers

Due to Faraday's law, a conductor that moves in a magnetic field is subject to a voltage (back electromotive force). If the conductor is shunted by a resistance the back electromotive force induces a current that interacts with the magnetic field and produces a Lorentz's force counteracting the motion of the conductor. As proposed by Karnopp [12], this principle can be simply implemented by connecting the electric terminals of a voice coil to a resistor. Even if voice coils are attractive for their simple configuration, they are characterized by a low force density. Torsional arrangements usually lead to lighter devices. If the objective is to damp a linear motion, the lighter weight of the torsional configuration may justify a mechanical device that transforms linear into torsional motion. The importance of the torsional configurations motivates the present study whose aim is to model a torsional damper and to study its dynamic behavior.

Fig. 1 shows the rotor of a torsional damper (or an induction machine) with one magnetic pole pair [14]. Although not represented in the figure, the stator generates a constant magnetic flux density $\mathbf{B}_s$. Two windings ($1, 1'$ and $2, 2'$) are installed on the rotor on orthogonal planes. The study is performed under the following assumptions:

- the electric parameters of the coils are the same (a sort of electric isotropy), and they are both shorted. The case of a generic passive shunt can be dealt with by considering the appropriate value of the impedance in series with the winding.
- The magnetic circuit is isotropic: the reluctance of the magnetic circuit is independent from the angular position. The results are then not applicable to reluctance motors and transformer dampers [6].
- The magnetic flux generated by the stator is constant as if it were produced by permanent magnets or by current driven electromagnets. Additionally, the magnetic field of the stator is fixed in space; this is equivalent to describe the system in a reference frame fixed to it.
- All quantities are independent from the axial coordinate.
- The rotor angular speed is given as input to the system; the torque being the corresponding output.

The arbitrary orientation of the currents shown in the figure does not affect the results. Unit vectors $\mathbf{n}_{r1}$ and $\mathbf{n}_{r2}$ are perpendicular to the plane of windings 1 and 2, respectively. Their orientation is obtained from the positive coil currents by the right-hand rule. Angles θ_1 of vector $\mathbf{n}_{r1}$ and θ_2 of $\mathbf{n}_{r2}$ are measured from the direction of the magnetic flux density $\mathbf{B}_s$. The rotation $\theta(t)$ of the rotor is the same as angle $\theta_1(t)$. Due to the orthogonality

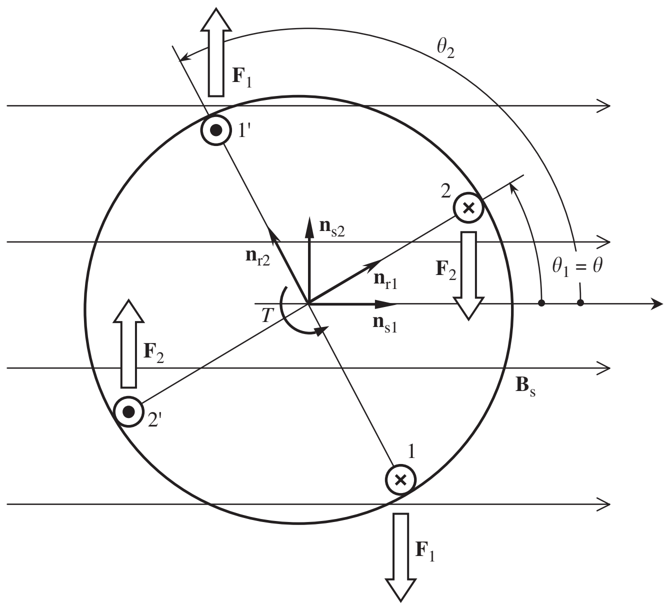

Fig. 1. Sketch of the induction machine. The stator generates a constant magnetic flux density vector $\mathbf{B}_s$. Two orthogonal windings are installed on the rotor. The diameter of the rotor is d, its length is l, the number of coils is N. $\mathbf{F}_1$ and $\mathbf{F}_2$ are the Lorentz forces acting on the conductors.

between the windings:

$$\theta_1(t) = \theta(t),$$
$$\theta_2(t) = \theta(t) + \pi/2. \tag{1}$$

The explicit dependence of the angles from time will be dropped in the following.

2.1. Flux linkages and currents

The total magnetic flux ϕ_{r1}, ϕ_{r2} linked by the rotor windings is due to the self-inductance and to the magnetic field of the stator

$$\phi_{ri} = L_r i_{ri} + \phi_{rsi}, \tag{2}$$

subscript $i = 1, 2$ indicates the two windings, and i_{ri} the currents in them. The magnetic fluxes ϕ_{rsi} generated by the stator (s) and linked by the rotor (r) can be obtained from the magnetic flux density $\mathbf{B}_s$, the number of coils N, the rotor length l, and diameter d

$$\phi_{rsi} = \int_{Si} \mathbf{B}_s \cdot \mathbf{n}\,\mathrm{d}S = Nld\mathbf{B}_s \cdot \mathbf{n}_{ri} = B_s Nld \cos\theta_i. \tag{3}$$

The integrals are computed on arbitrary surfaces S_1 and S_2 delimited by windings 1 and 2; $\mathbf{n}$ is the unit vector normal to the generic surface element $\mathrm{d}S$. Plane, rectangular surfaces are chosen for simplicity, a good alternative is to take them on the outer surface of the rotor. The term $B_s Nld$ represents the maximum magnetic flux linked by each winding (subscript 0)

$$\phi_{rs0} = B_s Nld. \tag{4}$$

Even if ϕ_{rs0} is constant, the rotor fluxes are modulated by the rotation of the rotor. Substituting Eqs. (1) in Eqs. (3) it follows:

$$\begin{aligned} \phi_{rs1} &= \phi_{rs0}\cos\theta_1 = \phi_{rs0}\cos\theta, \\ \phi_{rs2} &= \phi_{rs0}\cos\theta_2 = -\phi_{rs0}\sin\theta. \end{aligned} \tag{5}$$

Due to Faraday's law, the time dependence of angle θ causes an electromotive force. Eddy currents i_{ri} are induced in the windings. They are determined by Kirchoff's voltage law, taking the resistance R_r of the rotor windings into account

$$\frac{\mathrm{d}\phi_{ri}}{\mathrm{d}t} + R_r i_{ri} = 0. \tag{6}$$

The equations governing the currents are obtained substituting Eqs. (2) and (5) into Eq. (6)

$$L_r \frac{\mathrm{d}i_{r1}}{\mathrm{d}t} + R_r i_{r1} = \phi_{rs0}\sin\theta\dot{\theta},$$
$$L_r \frac{\mathrm{d}i_{r2}}{\mathrm{d}t} + R_r i_{r2} = \phi_{rs0}\cos\theta\dot{\theta}. \tag{7}$$

The electrical excitation of the rotor is due to the combined effects of the angular position (by means of angle θ) and velocity ($\dot{\theta}$). Integration of the above equations allows one to determine the coil currents as function of the time.

2.2. Electromechanical torque

The interaction between rotor currents i_{ri} and magnetic field of the stator generate Lorentz forces $\mathbf{F}_i$ on the conductors. The conductors and the magnetic field of Fig. 1, are perpendicular, so that:

$$F_i = B_s Nli_{ri}. \tag{8}$$

The electromagnetic torque T_{em} becomes

$$T_{\mathrm{em}} = -F_1 d\sin\theta - F_2 d\cos\theta = -\phi_{rs0}(i_{r1}\sin\theta + i_{r2}\cos\theta). \tag{9}$$

The magnetic flux of the stator ϕ_{rs0} converts the current in the electromagnetic torque, it can be interpreted as the torque constant of the electric machine (measured in Nm/A).

The torque generated by positive currents is opposite to rotation θ. An external torque $T = -T_{\text{em}}$ must be applied to the rotor to guarantee its equilibrium. Torque T_{em} of Eq. (9) can also be interpreted as the vector product ($\times$) between current vector $\mathbf{i}_r = i_{r1}\mathbf{n}_{r1} + i_{r2}\mathbf{n}_{r2}$ and magnetic flux vector ($\phi_{rs0} = \mathbf{B}_s Nld$)

$$\mathbf{T}_{\text{em}} = -\mathbf{i}_r \times \mathbf{B}_s Nld = -\mathbf{i}_r \times \phi_{rs0}. \tag{10}$$

Current vector $\mathbf{i}_r$ could be the resultant of a number of conductors distributed on the rotor. From this point of view at least two orthogonal windings per pole pair is necessary to allow the current vector span all rotor angles. As a limiting case, an infinity of evenly distributed conductors may be present. The case of a solid conductor is then equivalent to that of a discrete number of windings.

2.3. Equations in complex notation

Due to the circular symmetry of the system and to the orthogonality between the rotor windings, the flux and current vectors can be represented as complex quantities:

$$\begin{aligned} \phi_r &= \phi_{r1} + \mathrm{j}\phi_{r2}, \\ \phi_{rs} &= \phi_{rs1} + \mathrm{j}\phi_{rs2}, \\ i_r &= i_{r1} + \mathrm{j}i_{r2}, \end{aligned} \tag{11}$$

where $\mathrm{j} = \sqrt{-1}$. Considering Eqs. (5), the flux linkages due to the stator become,

$$\phi_{rs} = \phi_{rs0}\mathrm{e}^{-\mathrm{j}\theta}. \tag{12}$$

Similarly, the differential equations (7) can be rewritten in complex notation as

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = \mathrm{j}\phi_{rs0}\mathrm{e}^{-\mathrm{j}\theta}\dot{\theta}. \tag{13}$$

The angular velocity of the rotor ($\dot{\theta}$ of Eqs. (7)) and the electromechanical torque (T_{em} of Eq. (9)) are intrinsically real quantities. The last can be expressed as

$$T_{\text{em}} = -T = -\mathrm{Im}(\phi_{rs0} i_r \mathrm{e}^{\mathrm{j}\theta}). \tag{14}$$

Eqs. (13) and (14) constitute the system state and the output equations. Given angle θ as a function of the time, the electromechanical torque is obtained by first solving Eq. (13), and then substituting current i_r into Eq. (14).

3. Dynamic behavior

The aim of the following section is to characterize the dynamic behavior of eddy current dampers and induction machines in three modes of operation:

- *Motor/brake*: the rotor speed is constant relative to the magnetic field of the stator. This is typical of induction motors/generators and eddy current torsional couplers when no torsional vibration occurs.
- *Damper*: the rotor is subject to small amplitude torsional vibrations while its mean angular position is constant relative to the magnetic field of the stator. This mode is characteristic of torsional vibration dampers.
- *Coupler*: a constant angular speed is added to the torsional vibrations of the previous case. Eddy current torsional couplers operate in this mode when they transmit a mean torque and a torsional vibrations is occurring at the same time in the shaft.

3.1. Motor/brake

The motor/brake mode of operation is characterized by a constant, or slowly variable, angular speed Ω of the rotor relative to the magnetic field of the stator

$$\theta = \Omega t, \quad \dot{\theta} = \Omega. \tag{15}$$

An induction generator driven at constant speed by a turbine could be an example of the brake operation. A non-constant Ω can be considered as 'slowly variable' if its harmonic contributions are negligible relative to the R/L constant of Eq. (13). The aim of the present study is to obtain the torque as function of the angular speed (torque to speed characteristic). The following results are affected by the assumption that the magnetic flux generated by the stator is constant. What usually reported in the literature on the same subject is instead based on the assumption that the stator coils are voltage driven.

The differential equation governing the rotor currents i_r is obtained by substituting Eq. (15) in Eq. (13),

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = \mathrm{j}\phi_{rs0}\mathrm{e}^{-\mathrm{j}\Omega t}\Omega, \tag{16}$$

the steady-state solution is of the same type of the excitation

$$i_r = i_{r0}\mathrm{e}^{-\mathrm{j}\Omega t}, \tag{17}$$

where the complex amplitude i_{r0} is

$$i_{r0} = \frac{\mathrm{j}\phi_{rs0}\Omega}{R_r(1 - \mathrm{j}\Omega/\omega_p)}. \tag{18}$$

The pole frequency ω_p is the first-order dynamic of the two rotor coils,

$$\omega_p = \frac{R_r}{L_r}. \tag{19}$$

Considering Eq. (17), the current vector rotates in the complex plane at a speed $-\Omega$ relative to the rotor. At the same time the rotor runs with the angular speed Ω relative to the magnetic field of the stator, the current vector i_r is then fixed to the stator. The external torque (T) is obtained by substituting the induced current (Eq. (17)) in Eq. (14)

$$T = \frac{\phi_{rs0}^2}{R_r(1 + \Omega^2/\omega_p^2)}\Omega. \tag{20}$$

The external torque T and the rotating speed Ω have the same sign: mechanical power must then be given to the rotor to maintain its rotation.

To understand the dependence of the electromechanical torque from the angular speed it is useful to study the phase angle between the rotor currents and the magnetic field of the stator.

For small rotating speeds Ω relative to the pole frequency, the current vector in the rotor coordinate frame ($\mathbf{n}_{r1}$, $\mathbf{n}_{r2}$) is

$$i_r = \mathrm{j}\frac{\phi_{rs0}\Omega}{R_r}\mathrm{e}^{-\mathrm{j}\Omega t}, \quad \Omega \ll \omega_p. \tag{21}$$

In the stator coordinate frame ($\mathbf{n}_{s1}$, $\mathbf{n}_{s2}$) the same vector is

$$i_r^* = i_r\mathrm{e}^{\mathrm{j}p\Omega t}. \tag{22}$$

Taking the steady-state current of Eq. (17) into account, the current vector

$$i_r^* = \mathrm{j}\frac{\phi_{rs0}\Omega}{R_r} \tag{23}$$

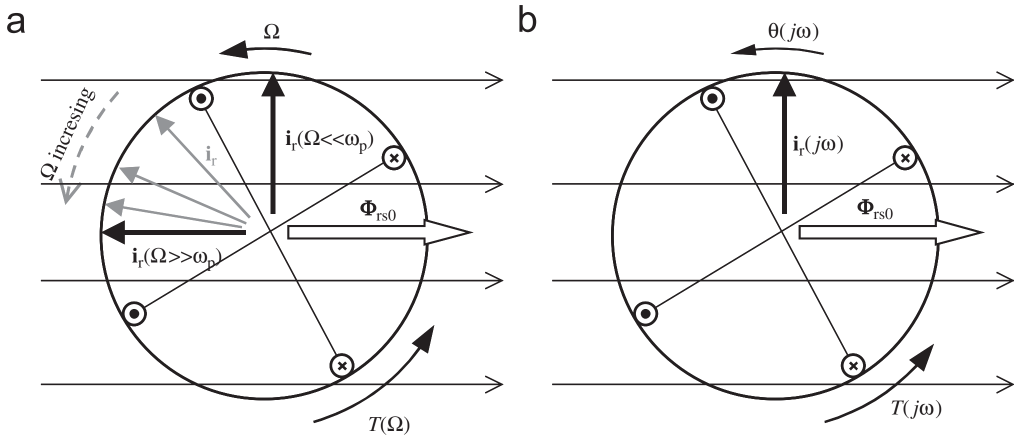

Fig. 2. Current ($\mathbf{i}_r$) and magnetic flux ($\boldsymbol{\phi}_{rs0}$) as function of the operating conditions. (a) Motor/brake operation: for $\Omega \ll \omega_p$ the current and the magnetic flux are perpendicular. For $\Omega \gg \omega_p$ (gray arrows) the current rotates and becomes opposite to the magnetic flux. (b) Damper operation: the current and the magnetic flux are always perpendicular, regardless of the frequency ω.

is fixed to the stator. The rotor current and the magnetic field are then in quadrature, as shown in the Fig. 2(a) ($i_r(\Omega \ll \omega_p)$). From Eq. (20), the torque is proportional to the rotating speed

$$T = c_0 \Omega \quad \text{where: } \Omega \ll \omega_p \text{ and } c_0 = \frac{\phi_{rs0}^2}{R_r}. \tag{24}$$

Eq. (24) shows that at speeds smaller than the pole frequency the device performs as a viscous joint of viscous coupling coefficient c_0.

By converse, if the speed is larger than the pole frequency, the current in the rotor windings is

$$i_r = -\frac{\phi_{rs0}}{L_r} e^{-j\Omega t} \quad \text{with } \Omega \gg \omega_p. \tag{25}$$

In the reference frame fixed to the stator the same current is constant

$$i_r^* = -\frac{\phi_{rs0}}{L_r}, \tag{26}$$

the negative sign indicates that the current is 180° out of phase relative to the magnetic field of the stator.

The path followed by the current vector for increasing speeds Ω is shown in Fig. 2(a) by gray arrows. For large rotating speeds the magnetic flux due to the rotor current counteracts that of the stator. In this speed range the torque decreases with the rotating speed Ω, so that the mechanical power ($T\Omega$) needed to run the rotor is constant

$$T = \frac{\phi_{rs0}^2}{R_r} \frac{\omega_p^2}{\Omega} = \frac{\phi_{rs0}^2}{L_r} \frac{\omega_p}{\Omega}. \tag{27}$$

The speed ($\Omega_{T_{\max}}$) corresponding to the maximum torque ($T_{\max}$) is obtained from Eq. (20),

$$\Omega_{T_{\max}} = \omega_p, \tag{28}$$

$$T_{\max} = \frac{\phi_{rs0}^2}{2L_r}. \tag{29}$$

From the comparison of Eqs. (20) and (24) it follows that the torque to speed characteristic is completely determined by the pole frequency ω_p and the viscous coupling coefficient c_0. For an existing machine these two parameters can be found by measuring the speed corresponding to the maximum torque ($\Omega_{T_{\max}}$), and the slope of the torque to speed characteristic close to the origin (c_0).

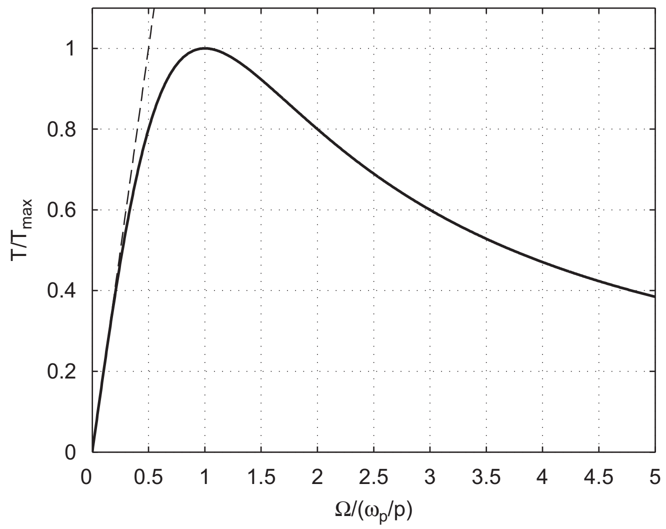

Fig. 3. Speed to torque characteristic of the brake (solid line). The dashed line represents the characteristic of a purely viscous damper (no inductive effect). $c_0 = 10\,\mathrm{Nms/rad}$, $\omega_p = 150\,\mathrm{rad/s}$.

Fig. 3 shows the torque to speed characteristic of an eddy current brake. The "electrical" speed $\Omega_e = p\Omega$ has been adopted instead of the rotor speed Ω for generality in order to take the number of pole pairs p into account. The electrical speed has then been made non-dimensional referring it to the pole frequency Ω_e/ω_p. Similarly, the torque has been made non-dimensional by referring it to its maximum value ($T/T_{\max}$).

If the usual definition of the slip factor s is taken into account,

$$s = \frac{\Omega_{mf} - \Omega_r}{\Omega_{mf}}, \tag{30}$$

the same curve can be converted into the torque to slip characteristic typical of asynchronous electric machines. Ω_{mf}, and Ω_r are the rotating speeds of the magnetic field and of the rotor, respectively.

3.2. Damper

The previous section shows that for (constant) angular speeds Ω smaller than pole frequency ω_p ($\Omega \ll \omega_p$), the torque to speed characteristic is linear with a good approximation; this suggests the use of the induction machine as torsional damper. The basic difference between the motor/brake operation and damper operation is that in the first case the relative speed between the rotor and the stator is constant while in the second it oscillates with a given frequency about a null mean value. To study the functionality as torsional damper in the following the rotor is assumed to vibrate with small amplitude $\theta_v(t)$ about an arbitrary mean angle θ_m:

$$\theta(t) = \theta_v(t) + \theta_m, \quad \theta_v(t) \ll 1. \tag{31}$$

The aim of the present section is to address the following two issues:

(1) to characterize the dynamic behavior of the eddy current damper in terms of its mechanical impedance, and
(2) to relate the mechanical impedance with the torque to speed characteristic as a motor/brake (Eq. (20) and Fig. 3). In other words: the induction machine "feels" only the instant value of the velocity or its behavior is determined by the frequency of the angular motion?

The assumption is that the amplitude of the angular speed is negligible compared to $\Omega_{T_{\max}}$ of Eq. (28). If the behavior of the induction machine were dominated only by the instant value of the speed, this assumption would imply that the output torque is proportional to the input speed, regardless of the angular frequency. By converse: if the frequency played a role, vibrations with the same angular speed amplitude but different frequencies would give a different response.

The currents in the rotor can be obtained substituting the angular position of Eq. (31) in Eq. (13),

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = \mathrm{j}\phi_{rs0}\mathrm{e}^{-\mathrm{j}\theta_v}\mathrm{e}^{-\mathrm{j}\theta_m}\dot{\theta}_v. \tag{32}$$

Due to the small angle assumption, the right-hand term of the differential equation can be simplified

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = \mathrm{j}\phi_{rs0}\mathrm{e}^{-\mathrm{j}\theta_m}\dot{\theta}_v. \tag{33}$$

The oscillation occurs about constant angle θ_m. Eq. (33) can be solved in the stator reference frame in terms of a current i_r^* rotated of the mean angle θ_m relative to i_r:

$$i_r^* = i_r \mathrm{e}^{\mathrm{j}\theta_m}. \tag{34}$$

The differential equation written in terms of i_r^* becomes

$$L_r \frac{\mathrm{d}i_r^*}{\mathrm{d}t} + R_r i_r^* = \mathrm{j}\phi_{rs0}\dot{\theta}_v. \tag{35}$$

As the angular speed $\dot{\theta}_v$ is a real quantity, current i_r^* is imaginary. The corresponding vector is then perpendicular to the magnetic flux density vector generated by the stator, as shown in Fig. 2(b). This is true regardless of the frequency content of the angular speed $\dot{\theta}_v$ and of mean angle θ_m. The damper mode is then substantially different than the motor/brake mode. As a matter of fact in the motor/brake mode the orientation of the current relative to the magnetic field of the stator is a function of the angular speed Ω.

The linearity of Eq. (35) allows one to analyze the system in terms of the transfer function between the input angular speed $\dot{\theta}$ and the output current i_r^*:

$$\frac{i_r^*(s)}{\dot{\theta}_v(s)} = \frac{\mathrm{j}\phi_{rs0}}{R_r(1 + s/\omega_p)}, \tag{36}$$

where s is the Laplace variable. The orthogonality between the current vector and the magnetic flux density of the stator ($\phi_{rs0}\mathbf{n}_{s1}$) allows then one to compute the torque acting on the rotor. The mechanical impedance $Z_m(s)$ is finally obtained as the ratio between the input speed and the output torque

$$Z_m(s) = \frac{T(s)}{\dot{\theta}_v(s)} = \frac{\phi_{rs0}^2}{R_r(1 + s/\omega_p)}. \tag{37}$$

The single pole, low-pass filter behavior of the mechanical impedance shows that for frequencies ($s = \mathrm{j}\omega$) lower then the pole, the device acts as a *linear viscous damper*. The related damping coefficient c_{em} is the same as the viscous coupling coefficient found in the case of the motor/brake operation (Eq. (24))

$$c_{\mathrm{em}} = \frac{T(\mathrm{j}\omega)}{\dot{\theta}_v(\mathrm{j}\omega)} = \frac{\phi_{rs0}^2}{R_r} = c_0, \quad \omega \ll \omega_p. \tag{38}$$

By converse, for frequencies much larger than the pole frequency, the torque is proportional to the relative displacement. The device acts as a *mechanical spring* of stiffness k_{em}

$$k_{\mathrm{em}} = \frac{T(\mathrm{j}\omega)}{\theta_v(\mathrm{j}\omega)} = \frac{\phi_{rs0}^2}{L_r}, \quad \omega \gg \omega_p. \tag{39}$$

Fig. 4 shows the amplitude of the mechanical impedance as function of the non-dimensional angular frequency ω/ω_p. The curve is obtained from Eq. (37) for the same device whose motor/brake operation is represented by the characteristic of Fig. 3.

The previous considerations yield that the mechanical impedance of an eddy current damper (Eq. (37)) is the same as a purely mechanical system constituted by the *series of a viscous damper* (*of damping coefficient* c_{em}) *and a spring* (*of stiffness* k_{em}). When the electromechanical damper is connected to a mechanical structure, the added electromechanical stiffness cannot be usually neglected. As a matter of fact, at frequencies higher than the pole it can give a remarkable contribution to overall stiffness of the structure.

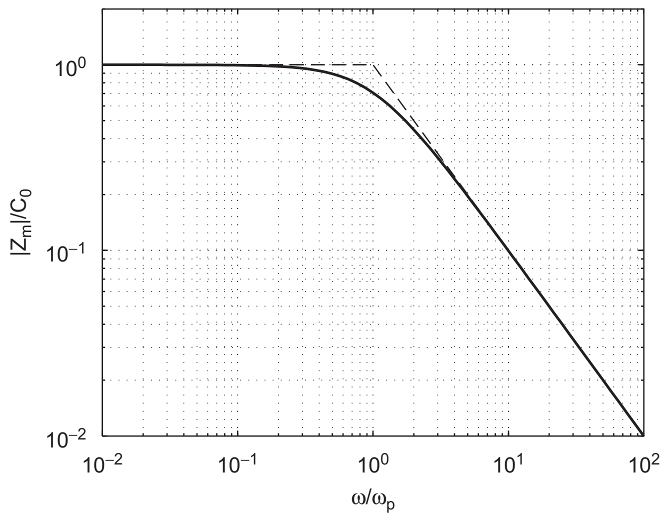

Fig. 4. Damper characteristic. $c_0 = 10\,\text{Nms/rad}$, $\omega_p = R_r/L_r = 150\,\text{rad/s}$. The mechanical impedance shows a first-order low-pass filter behavior, $\text{GBP} = c_0\omega_p = 1500\,\text{rad/s}$.

With reference to the mechanical impedance of Eq. (37), increasing the resistance R_r the frequency of the pole increases but the damping coefficient c_{em} decreases of the same amount. The torsional damper is then characterized by a gain bandwidth product (GBP) whose value is equal to the equivalent spring stiffness:

$$\text{GBP} = c_{\text{em}}\omega_p = \frac{\phi_{rs0}^2}{L_r} = k_{\text{em}}. \tag{40}$$

When the induction machine is to be used as damper, the rotor resistance must be designed as a trade off between the two contrasting needs of obtaining a high damping coefficient and that a wide enough bandwidth. If the compromise cannot be reached, the solution could be to try a larger gain-bandwidth product (GBP). This could require either (1) a reduction of the rotor inductance (L_r) or (2) an increment of the magnetic flux linkage ϕ_{rs0} generated by the stator. This is the same as increasing the current constant of the electrical machine. The gain-bandwidth product is then a basic indicator of the overall dynamic performances of an eddy current damper. It can be shown that it is a function of the size of the device and the amount (and type) of permanent magnets.

Although the band limited nature of the damping may be a drawback for some applications, it can be beneficial for vibration isolation. The pole frequency can be tuned to give high damping in a low frequency range including the mechanical resonances, and to give a purely elastic contribution at high frequency (vibration isolation range), where a non-band limited device would increase the transmissibility.

3.3. Coupler

In the previous sections the study was focused on the behavior of the induction machine of Fig. 1 for the cases of constant angular speed and small amplitude torsional vibrations. The aim of the present section is to analyze the intermediate case: i.e. the superposition of a constant rotating speed Ω with small amplitude vibrations. This condition occurs when induction motors, brakes or couplers are connected to a non-constant load. From a different point of view, the objective is to determine the mechanical impedance while the induction machine is transmitting mechanical power. To simplify the analysis the vibrations are assumed to by of harmonic type of (small) amplitude θ_0 and angular frequency ω. If at time $t = 0$ the stator and the rotor are aligned, the angle at time t is

$$\theta = \Omega t + \text{Re}(\theta_0 e^{j\omega t}) = \Omega t + \theta_0 \cos \omega t = \Omega t + \theta_\omega, \quad \theta_0 \ll 1. \tag{41}$$

The angular speed is then

$$\dot{\theta} = \Omega + \theta_0 \,\mathrm{Re}(\mathrm{j}\omega \mathrm{e}^{\mathrm{j}\omega t}) = \Omega - \theta_0 \omega \sin \omega t = \Omega + \dot{\theta}_\omega. \tag{42}$$

The rotor currents are found by substituting the angular position and velocity of Eqs. (41) and (42) in Eq. (13)

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = \mathrm{j}\phi_{rs0} \mathrm{e}^{-\mathrm{j}\theta} \dot{\theta} = \mathrm{j}\phi_{rs0} \mathrm{e}^{-\mathrm{j}\Omega t} \mathrm{e}^{-\mathrm{j}\theta_0 \cos \omega t} \dot{\theta}. \tag{43}$$

Due to the small value of angle θ_0, the excitation voltage can be split in two contributions

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = V_\Omega + V_{\omega\Omega}. \tag{44}$$

The first excitation voltage (V_Ω) is the same as for the motor/brake operation (Eq. (16)). The second ($V_{\omega\Omega}$) is due to the combination of slip speed Ω and angular frequency ω:

$$V_\Omega = \mathrm{j}\phi_{rs0} \mathrm{e}^{-\mathrm{j}\Omega t} \Omega, \tag{45}$$

$$V_{\omega\Omega} = \phi_{rs0} \theta_0 (\Omega \cos \omega t - \mathrm{j}\omega \sin \omega t) \mathrm{e}^{-\mathrm{j}\Omega t}. \tag{46}$$

By exploiting the relations for the product of trigonometric functions, voltage $V_{\omega\Omega}$ can be expressed in terms of sum and difference of rotation speed Ω and oscillation frequency ω:

$$V_{\omega\Omega} = \frac{\phi_{rs0}\theta_0}{2} [(\Omega - \omega) \mathrm{e}^{-\mathrm{j}(\Omega-\omega)t} + (\Omega + \omega) \mathrm{e}^{-\mathrm{j}(\Omega+\omega)t}]. \tag{47}$$

The linearity of Eq. (44) allows one to compute the overall response as the superposition of the responses to each excitation voltage

$$i_r(t) = i_{r0} \mathrm{e}^{-\mathrm{j}\Omega t} + i_{r\omega\Omega}(t). \tag{48}$$

The first term is the same as for the motor/brake (Eqs. (17) and (18)). The second ($i_{r\omega\Omega}(t)$) is due to voltage $V_{\omega\Omega}$. It is represented by a vector rotating at an average angular speed $-\Omega$ relative to a frame fixed to the rotor. This vector is then fixed to the stator. After the transient due to the initial conditions dies out, the steady-state solution becomes

$$i_{r\omega\Omega}(t) = \frac{\phi_{rs0}\theta_0}{2R_r} \left(\frac{\Omega - \omega}{1 - \mathrm{j}(\Omega - \omega)/\omega_p} \mathrm{e}^{-\mathrm{j}(\Omega-\omega)t} + \frac{\Omega + \omega}{1 - \mathrm{j}(\Omega + \omega)/\omega_p} \mathrm{e}^{-\mathrm{j}(\Omega+\omega)t} \right). \tag{49}$$

The electromechanical torque is computed from Eq. (14). Resorting to the small angle assumption for amplitude θ_0 this leads to

$$\begin{aligned} T &= \phi_{rs0} \,\mathrm{Im}(i_r(t) \mathrm{e}^{\mathrm{j}(\Omega t + \theta_0 \cos \omega t)}) = \phi_{rs0} \,\mathrm{Im}[i_r \mathrm{e}^{\mathrm{j}\Omega t}(1 + \mathrm{j}\theta_0 \cos \omega t)] \\ &\approx \phi_{rs0} \,\mathrm{Im}[(i_{r0} + i_{r\omega\Omega}(t) \mathrm{e}^{\mathrm{j}\Omega t})] = T_\Omega + T_{\omega\Omega}(t). \end{aligned} \tag{50}$$

Similarly to the current, also the torque is the sum of a constant contribution (T_Ω) due to the average slip speed Ω, and of an harmonic contribution ($T_\omega(t)$) due to the oscillation about that speed. The first is the same as for the motor/brake (Eq. (20)), the second is

$$\begin{aligned} T_{\omega\Omega}(t) &= \phi_{rs0} \mathrm{Im}(i_{r\omega\Omega}(t) \mathrm{e}^{\mathrm{j}\Omega t}) \\ &\approx \frac{\phi_{rs0}^2}{2R_r} \mathrm{Im} \left[\frac{\Omega - \omega}{1 - \mathrm{j}(\Omega - \omega)/\omega_p} \mathrm{e}^{\mathrm{j}\omega t} + \frac{\Omega + \omega}{1 - \mathrm{j}(\Omega + \omega)/\omega_p} \mathrm{e}^{-\mathrm{j}\omega t} \right] \theta_0. \end{aligned} \tag{51}$$

The torque $T_{\omega\Omega}(t)$ can be studied in a number of cases to understand the behavior of the coupler for what the oscillation components of the angular speed is concerned.

1st case: $\Omega \ll \omega_p$.

For angular speeds Ω smaller than the pole frequency ω_p, it is necessary to analyze two cases depending on the value of the oscillation frequency ω relative to pole frequency ω_p.

If $\Omega \ll \omega_p$, and $\omega \ll \omega_p$;

If both oscillation frequency and angular speed Ω are smaller than the pole frequency ω_p, the torque is proportional to the rotor oscillation speed

$$T_{\omega\Omega} \approx \frac{\phi_{rs0}^2}{2R_r}\mathrm{Im}[(\Omega-\omega)\mathrm{e}^{\mathrm{j}\omega t}+(\Omega+\omega)\mathrm{e}^{-\mathrm{j}\omega t}]\theta_0$$
$$= -\frac{\phi_{rs0}^2}{R_r}\omega\theta_0\sin\omega t = \frac{\phi_{rs0}^2}{R_r}\dot{\theta}_\omega. \tag{52}$$

This is the same as for the damper operating at low frequency (Eq. (38)). Putting together the responses to constant and vibration speeds,

$$T = c_{\mathrm{em}}(\Omega + \dot{\theta}_\omega), \tag{53}$$

the response is then of viscous type for both components.

Else, if $\Omega \ll \omega_p$, and $\omega \gg \omega_p$;

If the vibration frequency ω is larger than the pole frequency (ω_p), and both are larger than angular speed Ω,

$$T_{\omega\Omega} \approx \frac{\phi_{rs0}^2}{2R_r}\mathrm{Im}\left(-\frac{\omega_p}{\mathrm{j}}\mathrm{e}^{\mathrm{j}\omega t}-\frac{\omega_p}{\mathrm{j}}\mathrm{e}^{-\mathrm{j}\omega t}\right)\theta_0$$
$$= \frac{\phi_{rs0}^2}{R_r}\omega_p\theta_0\cos\omega t = \frac{\phi_{rs0}^2}{L_r}\theta_0\cos\omega t = k_{\mathrm{em}}\theta_\omega, \tag{54}$$

the torque $T_{\omega\Omega}$ due to vibration is proportional to the rotation (θ_ω from Eq. (41)). Similarly to the previous case, the response to vibrations is the same as for the damper mode (Eq. (39)),

$$T = c_{\mathrm{em}}\Omega + k_{\mathrm{em}}\theta_\omega. \tag{55}$$

As a summary of the case, when the constant component (Ω) of the rotating speed is smaller than the pole frequency, the coupler behaves relative to the vibration as a eddy current damper regardless of the vibration frequency.

2nd case: $\Omega \gg \omega_p$.

The second case occurs when the angular speed is higher than the pole frequency ($\Omega \gg \omega_p$). From Eq. (51), the torque due to rotor oscillations is that of Eq. (54). This is as if the oscillation frequency were higher than the pole. The total torque is

$$T = k_{\mathrm{em}}\left(\frac{\omega_p}{p\Omega}+\theta_\omega\right). \tag{56}$$

Regardless the frequency of the oscillation, the torque decreases with the inverse of the constant part of the angular speed Ω, and is proportional to the vibration angle θ_ω.

4. Influence of the number of magnetic pole pairs

The number of magnetic pole pairs is one of the main design parameters of eddy current dampers and couplers. The objective is now to determine how the torque to speed characteristic and the frequency response are affected by the number of pole pairs. As the cases of one, and p pole pairs are very similar, only the peculiar aspects of the last are considered in detail.

The radial component of the magnetic flux density of the stator is assumed to make p complete cycles around the stator. With $\mathbf{n}$ indicating the normal to the rotor surface,

$$\mathbf{B}_s \cdot \mathbf{n} = B_s\cos(p\beta). \tag{57}$$

The description angle β indicates an angular coordinate on the stator. In the one pole pair system of Fig. 1, the normal field $\mathbf{B}_s \cdot \mathbf{n}$ completes one cycle for β spanning the whole stator ($0<\beta<2\pi$). Correspondingly, two equally spaced rotor windings are linked to the magnetic field. Each winding spans an angle ($\Delta\beta = \pi$ rad) that is half the period of the magnetic field of the stator.

Similarly, in the case of p pole pairs, for $0<\beta<2\pi$ the normal field $\mathbf{B}_s \cdot \mathbf{n}$ makes p cycles, and $2p$ windings are installed on the rotor. Each winding spans a rotor angle $\Delta\beta = \pi/p$ rad. Similarly to what is done in Eq. (3) for a single pole pair, the stator flux linked to the ith coil is

$$\phi_{rsi} = \int_{Sii'} \mathbf{B}_s \cdot \mathbf{n}\,\mathrm{d}S = \int_{\theta_i-\pi/2p}^{\theta_i+\pi/2p} B_s \cos(p\beta) Nl \frac{\mathrm{d}}{2} d\beta = \frac{\phi_{rs0}}{p}\cos(p\theta_i). \tag{58}$$

Subscript $i = 1, 2$ indicates the two windings corresponding to each pole pair, and ϕ_{rs0} is the same as given in Eq. (4). The electromechanical torque T_{em} is given by the contributions of the p pole pairs. They are computed from the flux linkages (instead of from the Lorentz force) as this avoids the explicit computation of the directions of the forces acting on the coils

$$T_{\mathrm{em}} = -p\left(i_{r1} \frac{\partial \phi_{rs}}{\partial \theta}_{\theta=\theta 1} + i_{r2} \frac{\partial \phi_{rs}}{\partial \theta}_{\theta=\theta 2} \right). \tag{59}$$

By substituting Eq. (58) into Eq. (59) the latter yields

$$T_{\mathrm{em}} = -p\phi_{rs0}(i_{r1}\sin\theta_e + i_{r2}\cos\theta_e) = -\phi_{rs0}\,\mathrm{Im}(i_r \mathrm{e}^{\mathrm{j}\theta_e}). \tag{60}$$

The electrical angle θ_e is defined as $\theta_e = p\theta$. Resorting to the complex coordinates, the total flux linked by each coil is

$$\phi_r = L_r i_r + \phi_{rs} = L_r i_r + \frac{\phi_{rs0}}{p}\mathrm{e}^{-\mathrm{j}\theta_e}. \tag{61}$$

The equation governing the currents becomes,

$$L_r \frac{\mathrm{d}i_r}{\mathrm{d}t} + R_r i_r = \mathrm{j}\phi_{rs0}\mathrm{e}^{-\mathrm{j}\theta_e}\dot{\theta}. \tag{62}$$

For a given rotor speed, the electromechanical torque (T_{em}) is obtained substituting the currents i_r obtained from Eq. (62) in Eq. (60). The torque to speed characteristic for constant speed Ω, and the mechanical impedance result to be

$$T(\Omega) = \frac{\phi_{rs0}^2}{R_r(1 + (p\Omega)^2/\omega_p^2)} p\Omega, \tag{63}$$

$$Z_m(s) = \frac{p\phi_{rs0}^2}{R_r(1 + s/\omega_p)}, \tag{64}$$

where the $\omega_p = R_r/L_r$. The number of pole pairs affects the torque to speed characteristic of Eq. (63) as if the effective speed were the electrical speed $\Omega_e = p\Omega$. This speed is the excitation frequency of each winding. For what the frequency behavior is concerned, the mechanical impedance of Eq. (64) is not affected by the number of poles. This is because for small amplitude vibrations electrical excitation frequency and mechanical frequency are the same.

To conclude this section, note that the resistance and inductance (R_r and L_r) are implicit functions of the number of pole pairs. This dependence is not investigated here even if it represents a very important design issue.

5. Motor/brake–damper

The considerations about the torque to speed characteristic and the mechanical impedance show that the design of an eddy current coupler is subject to rather contrasting needs relative to the design of a torsional damper.

- *Coupler*: A low value of the resistance allows one to obtain a high damping coefficient (c_0) and a low pole frequency (ω_p). This is convenient for a coupler as the high value of c_0 allows transmitting the mechanical power with low slip and, consequently, a small dissipation. Additionally, the lower is the value of the pole

frequency, the better the load is isolated from the torsional vibrations. Due to the low resistance requirement, copper conductors are better suited than aluminum ones for this application.
- *Damper*: If high-frequency damping is required, an additional shunt resistor, or a conductor with higher resistance, could be useful in order to let the electromechanical device work below the pole frequency. If solid conductors are installed on the rotor, the use of aluminum instead of copper can be expedient to this end. Additionally, if considering the same volume of conductor, the lower mass density of the aluminum (compared to the copper) allows one to reduce the added mass properties of the rotor. This may improve the dynamic behavior of the system. It may be of little or no help to reduce the rotor resistance with the aim of increasing the low-frequency damping. This increment is in fact obtained at the cost of a lower bandwidth.

In the design practice the torque to speed characteristic is usually available for constant speed operation (motor/brake) either by means of a direct measurement or by simulation using software tools. The mechanical impedance can be seldom obtained from the same software tools and/or experimental setup and usually requires time domain simulations or a dedicated characterization. The previous study shows that the torque to speed characteristic of Eq. (63) and the mechanical impedance of Eq. (64) are affected by the same parameters and may be rewritten as

$$T(\Omega) = \frac{c_0}{1 + (p\Omega)^2/\Omega_{T_{\max}}^2}\Omega, \tag{65}$$

$$Z_m(s) = \frac{c_{\mathrm{em}}}{1 + s/\omega_p}. \tag{66}$$

The slope of the torque to speed characteristic for low speed (c_0) and the damping for low frequency (c_{em}) are the same

$$Z_m(\mathrm{j}\omega = 0) = c_{\mathrm{em}} = c_0 = \left.\frac{\mathrm{d}T(\Omega)}{\mathrm{d}\Omega}\right|_{\Omega=0}. \tag{67}$$

The pole frequency (ω_p), the gain-bandwidth (GBP) and the electromechanical stiffness (k_{em}) are related to the angular speed ($\Omega_{T_{\max}}$) corresponding to the maximum torque $T_{\max}$:

$$\omega_p = \frac{k_{\mathrm{em}}}{c_{\mathrm{em}}} = p\Omega_{T_{\max}}, \tag{68}$$

$$k_{\mathrm{em}} = \mathrm{GBP} = 2pT_{\max}. \tag{69}$$

Eqs. (67)–(69) allow one to obtain one characteristic from the other by measuring a limited number of characteristic points of the curves. This may lead to a considerable reduction of the computational/ experimental effort during the characterization of an induction machine.

6. Conclusions

The electromechanical model of eddy current dampers and couplers constituted by a conductor and a magnetic circuit in relative motion has been developed starting from Lorentz's and Faraday's laws. This kind of eddy current device is indicated of the "motional" type to distinguish it from that based on variable reluctance effects ("transformer" type). The model is written in terms of state and output equations, the input being the relative speed between the rotor and the stator, the mechanical torque being the corresponding output. The damper model can be interfaced to that of a generic mechanical system to study the effects on the system behavior.

The generic equations are solved for constant speed and for torsional oscillations at a given frequency. For constant speed between the stator and the rotor the eddy current device operates as a coupler and is described by the torque to speed characteristic. This characteristic is similar to that of induction motors, for low speeds is almost linear showing a viscous behavior, for high speeds the torque decreases with increasing speed.

For small amplitude mechanical oscillations the behavior is described by the mechanical impedance. This characteristic shows a first-order low-pass filter behavior. Below the pole the torque is proportional to the angular speed, in that frequency range the damper behaves as a viscous damper. Above the pole the torque is proportional to the angular displacement, in that range the damper behaves as a mechanical spring. The pole frequency can be adapted by modifying the shunt resistance in series with the conductor, the higher the shunt resistance the higher the pole frequency. Nevertheless the increment of the pole frequency is obtained at the cost of lowering the low-frequency damping of the same amount. The mechanical impedance is therefore characterized by a constant product between the low-frequency damping coefficient and the pole frequency (gain bandwidth product). The gain bandwidth product is not a function of the shunt resistance but only of the magnetic flux linkage and of the coil inductance. The low-pass filter behavior of the mechanical impedance suggests a mechanical analogue given by the series of a purely viscous damper and a spring. This analogy may be useful for understanding the interaction between a motional eddy current damper and a mechanical structure.

Although the torque to speed characteristic for constant speed and the mechanical impedance are conceptually different characteristics, they are affected by the same parameters. The relationship between them has been reported explicitly taking the number of pole pairs into account. This allows to determine the mechanical impedance from the measurement of the torque to speed characteristic and vice versa.

The modeling approach is not only applicable to electric machines based on permanent magnets but also to current driven induction machines. It can then be used to study the torsional dynamic behavior of transmission shafts including an induction machine of the motor, generator or coupling type.

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